12 MATHEMATICS METHODS
COMMON TEST 3 – Term 1 2016
Integration Techniques

Name: Solutions Marks: / 43

Instructions:
- External notes are not allowed
- Duration of test: 40 minutes
- This test contributes to 6% of the year (school) mark
- No calculator

Full marks may not be awarded to correct answers unless sufficient justification is given.

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1. (3 marks)

Evaluate $\int_{-\pi}^{\pi} \cos(x/2)dx$

$= \left[\frac{\sin(\frac{x}{2})}{1/2} \right]_{-\pi}^{\pi}$ ✓ anti derivative

$= \left(2\sin\left(\frac{\pi}{2}\right)\right) - \left(2\sin\left(\frac{-\pi}{2}\right)\right)$ ✓ FTC

$= 2 - (-2)$

$= 4$ ✓ answer

2. **(8 marks)**

Determine the following integrals:

(a) $\int \frac{3}{x^{-2}} + 4\,dx$ [2]

$= \int 3x^2 + 4\,dx$

$= x^3 + 4x + c$

anti derivative

(b) $\int \frac{(4-x)}{\sqrt{x}}\,dx$ [3]

$= \int \frac{4}{\sqrt{x}} - \frac{x}{\sqrt{x}}\,dx$ divide

$= \int 4x^{-1/2} - x^{1/2}\,dx$

$= 8x^{1/2} - \frac{x^{3/2}}{3/2} + c$

$= 8\sqrt{x} - \frac{2}{3}\sqrt{x^3} + c$

anti derivative

(c) $\int \frac{1}{(2x-1)^5}\,dx$ [3]

$= \int (2x-1)^{-5}\,dx$

$= \frac{(2x-1)^{-4}}{(-4)(2)} + c$

$= \frac{-1}{8(2x-1)^4} + c$

coeff bracket

-1 mark overall this page no + c

3. **(7 marks)**

Evaluate

(a) $\frac{d}{dx}\int_{-4}^{x}\sqrt{5t^2-3}\,dt$

[1]

$= \sqrt{5x^2-3}$ ✓ derivative of the integral

(b) $\frac{d}{dx}\int_{-1}^{-x^3}\frac{t}{(t-2)^2}\,dt$

✓ $f(g(x))$ ✓ $g'(x)$

$= \frac{-x^3}{(-x^3-2)^2} \times (-3x^2)$

$= \frac{3x^5}{(-x^3-2)^2}$ ✓ simplify

[3]

(c) $\int_{2x}^{1}\frac{d}{dt}\left[t\sqrt{1+t^2}\right]dt$

$= \left[t\sqrt{1+t^2}\right]_{2x}^{1}$ integral of the derivative ✓

[3]

$f(b) - f(a)$

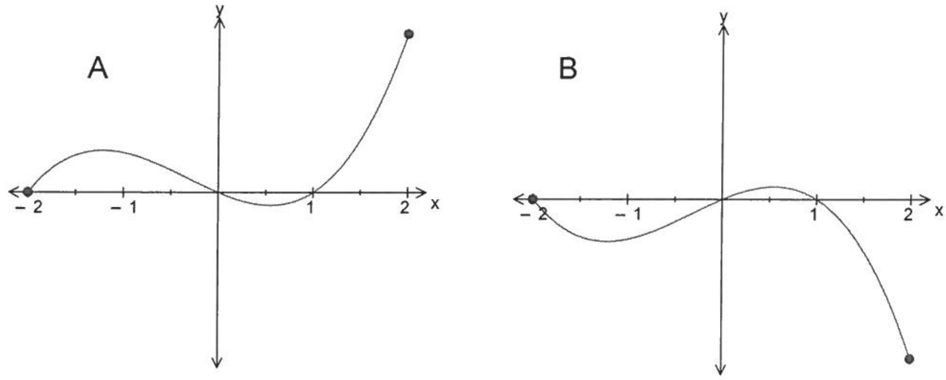
$= \sqrt{2} - 2x\sqrt{1+4x^2}$ ✓ ✓

4. (5 marks)

Two functions $f(x)$ and $g(x)$ exist such that:

$$\int_{-2}^{0} f(x)\,dx = 2 \qquad \text{and} \qquad \int_{1}^{0} g(x)\,dx = -1$$

(a) Determine which of the following graphs are $f(x)$ and $g(x)$. [2]



f(x) ✓ g(x) ✓

(b) Answer true or false for each of the following. [3]

(i) $\int_{-2}^{2} f(x)\,dx > \int_{-2}^{2} g(x)\,dx$ T ✓

(ii) $\int_{0}^{2} f(x)\,dx > \int_{-2}^{2} f(x)\,dx$ F ✓

(iii) $\int_{-2}^{2} g(x)\,dx > 0$ F ✓

5. **(3 marks)**

The gradient function of a curve is given by $\frac{dy}{dx} = x^2 - 4e^{-2x}$

Find the equation of this curve given it passes through the point $(0\ ,3)$

$$y = \frac{x^3}{3} - \frac{4e^{-2x}}{-2} + c$$

✓ anti derivative

$$y = \frac{x^3}{3} + 2e^{-2x} + c$$

$(0,3)$ $\quad 3 = 0 + 2 + c$

$c = 1$ ✓ find c

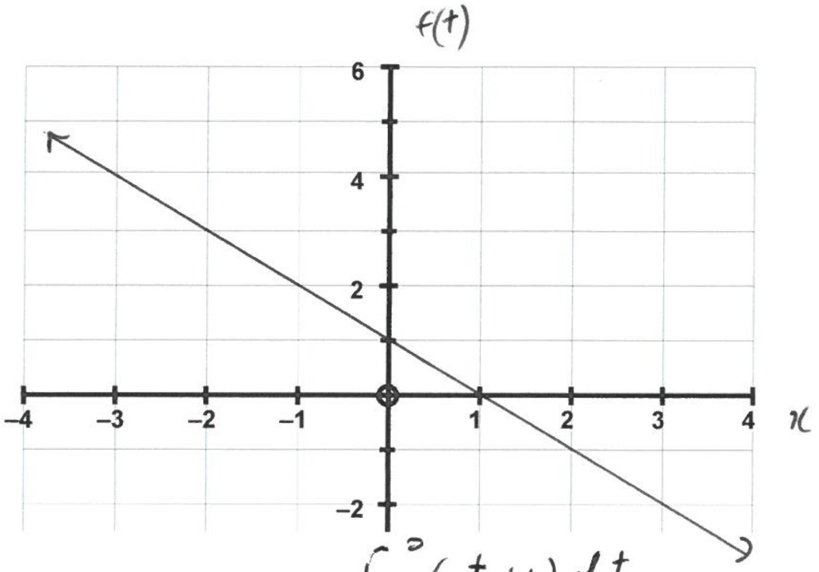
$$\therefore \quad y = \frac{x^3}{3} + 2e^{-2x} + 1$$

✓ equation of curve

7 **(5 marks)**

Consider $A(x) = \int_{-1}^{x} (-t+1)\,dt$

Plot $f(t) = -t+1$



Area function

Accumulation function

(a) Find

$A(-1) = \int_{-1}^{-1} (-t+1)\,dt$

$= 0$

$A(0) = \int_{-1}^{0} (-t+1)\,dt$

$= 1\frac{1}{2}$

$A(1) = \int_{-1}^{1} (-t+1)\,dt$

$= 2$

$A(2) = 1\frac{1}{2}$ [2]

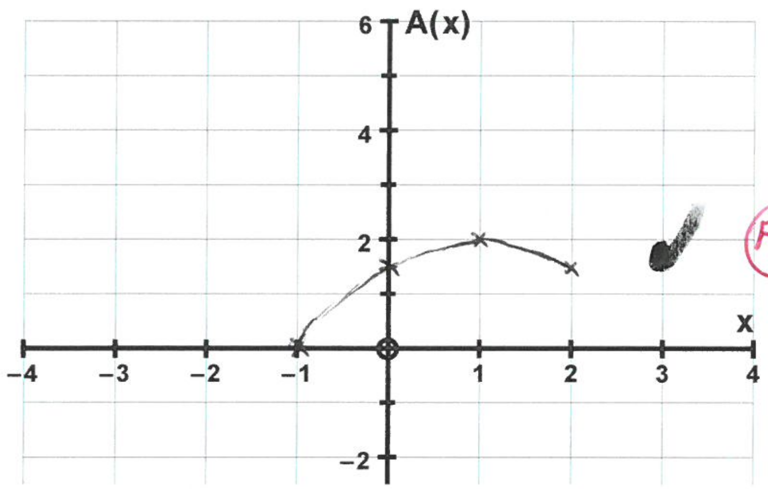
all correct 2 marks

2 correct 1 mark

(no ½ marks)

(b) Plot the values in part (a) and hence sketch the graph of $A(x)$ for $-1 \le x \le 2$

[1]



FT

(c) Determine the defining rule for (i) $A'(x) = -x+1$

(ii) $A(x) = -\frac{x^2}{2} + x + 1.5$ [2]

or

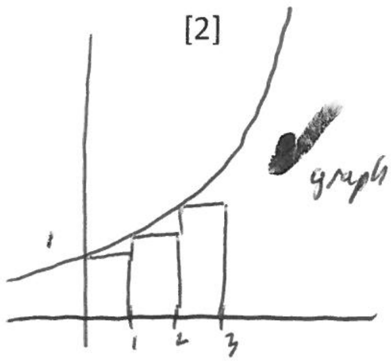
$A(x) = -\frac{1}{2}(x-1)^2 + 2$

6. (7 marks)

(a) Find an approximation to the area of the region between

$y = e^{2x}$, and the lines x = 0, x = 3 and the x axis using exact values and

(i) 3 left rectangles [2]



$$\text{Area} = 1 \times (e^0 + e^2 + e^4)$$
$$= 1 + e^2 + e^4 \text{ units}^2$$

area

(ii) 3 right rectangles [1]

$$\text{Area} = 1 \times (e^2 + e^4 + e^6)$$
$$= e^2 + e^4 + e^6 \text{ units}^2$$

area

(iii) The average of parts (i) and (ii) [1]

$$\frac{1 + e^2 + e^4 + e^2 + e^4 + e^6}{2} = \frac{1 + 2e^2 + 2e^4 + e^6}{2}$$

264.2

ave

(b) Evaluate using exact values $\int_0^3 e^{2x}\,dx$ [2]

$$= \left[\frac{e^{2x}}{2}\right]_0^3$$

anti derivative

$$= \left(\frac{e^6}{2}\right) - \left(\frac{e^0}{2}\right)$$
$$= \frac{e^6}{2} - \frac{1}{2}$$

FTC

201.2

(c) Explain why the answers in parts (a) and (b) are different. [1]

Part (a) uses only 3 rectangles and part (b) uses

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,\delta x_i \text{ where } \delta x \to 0 .$$

idea

8. **(5 marks)**

(a) Find $\frac{dy}{dx}$ given that $y = x\cos x$ [2]

$vu' + uv'$

$$\frac{dy}{dx} = \cos x \cdot 1 + x(-\sin x)$$

$$= \cos x - x\sin x$$

product rule

(b) Use your answer in part (a) to find $\int (x\sin x)dx$ [3]

integral of the derivative

$$\int \cos x - x\sin x \; dx = x\cos x + c$$

$$\int \cos x \, dx - \int x\sin x \, dx = x\cos x + c$$

$$\sin x - \int x\sin x \, dx = x\cos x + k$$

$$\int x\sin x \, dx = \sin x - x\cos x + k$$

answer